The $\bar{B} \to D^* \ell \bar{\nu}$ semileptonic decay at non-zero recoil from the Fermilab/MILC collaborations

Alejandro Vaquero

University of Utah

December 16th, 2019

On behalf of the Fermilab/MILC collaborations, with:

Carleton DeTar, University of Utah
Aida El-Khadra, University of Illinois
Andreas Kronfeld, FNAL
John Laiho, University of Syracuse
Ruth Van de Water, FNAL
The Standard Model (SM)

- The Standard Model is (arguably) the most successful theory describing nature we have ever had.
- The theory is not completely satisfactory.
  - Situation similar to that at the end of the XIX century.
- The SM can explain phenomena in a large range of scales.

Yet there is a region where we expect the SM to fail.
- The SM is regarded as an effective theory at low energies (low means $E \lesssim v_{EW} \approx 0.1 - 1$ TeV).
Where to look for new physics?

Energy frontier

Intensity frontier

Cosmology frontier
Flavor physics and the CKM matrix

- The CKM matrix is just a basis transformation

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\]

- Matrix must be unitary (preserve the norm)

- Unitarity imposes several constraints

\[
|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 =
|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1
\]

- The \( V_{ub}/V_{cb} \) ratio determines the opposite side to the angle \( \beta \) in the unitarity triangle
Long standing tension

Latest PDG values show no tension, but this is not the current consensus

| Determination             | $|V_{cb}| (\cdot 10^{-3})$ |
|---------------------------|--------------------------|
| Exclusive (PDG’16)        | 39.2 ± 0.7               |
| Inclusive (PDG’16)        | 42.2 ± 0.8               |
| Exclusive (PDG’18)        | 41.9 ± 2.0               |
| Inclusive (PDG’18)        | 42.2 ± 0.8               |

$\chi^2$/dof = 42.3/23 (CL = 0.84 %)
The $V_{cb}$ matrix element: Measurement from exclusive processes

- Experiments measure the decay rate as a function of $w = v_{D^*} \cdot v_B$

$$
\frac{d\Gamma}{dw} (\bar{B} \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^2} |V_{cb}|^2 (w^2 - 1)^{\frac{1}{2}} P(w) |\eta_{ew} F(w)|^2
$$

- Exclusive determination
- Efficient decay and well understood
- Extensively studied in $B$ experiments (BaBar, CLEO, LHCb, Belle I and II...)
The $V_{cb}$ matrix element: Measurement from exclusive processes

\[ \frac{d\Gamma}{dw} (\bar{B} \to D^* \ell \bar{\nu}_\ell) = \frac{G_F^2 m_B^5}{48\pi^2} (w^2 - 1)^{\frac{1}{2}} P(w) |\eta_{ew}|^2 |\mathcal{F}(w)|^2 |V_{cb}|^2 \]

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative

Heavy Quark Effective Theory

- Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
- $\lim_{m_Q \to \infty} \mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
- **We don’t know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
- At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O \left( \alpha_s, \frac{\Lambda_{QCD}}{m_Q} \right)$
- An old estimate gave $\mathcal{F}(1) = 0.907 \pm 0.026$

- Reduction in the phase space $(w^2 - 1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
- Need to extrapolate $|V_{cb}|^2 |\eta_{ew}\mathcal{F}(w)|^2$ to $w = 1$
- This extrapolation is done using well established parametrizations
The $V_{cb}$ matrix element: The parametrization issue

All the parametrizations perform an expansion on the $z$ parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- **Boyd-Grinstein-Lebed (BGL)**

$$f_X(w) = \frac{1}{B_{fX}(z)\phi_{fX}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - Includes unitarity constrains $\sum_n |a_n|^2 \leq 1$ and poles through the Blaschke factors $B_{fX}(z)$
  
- **Caprini-Lellouch-Neubert (CLN)**

  $$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with} \ c = f_c(\rho), \ d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$
The $V_{cb}$ matrix element: The parametrization issue

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one
  \[ |V_{cb}| = 41.7 \pm 2.0(\times 10^{-3}) \]

Latest Belle dataset and Babar analysis seem to contradict this picture
- From Babar’s paper arXiv:1903.10002 BGL is compatible with CLN and far from the inclusive value
- Belle’s paper arXiv:1809.03290v3 finds similar results in its last revision

The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$
The $V_{cb}$ matrix element: The parametrization issue

- The CLN parametrization strongly constrains the relations between the different coefficients of the $z$-expansion.

- The original CLN paper provides errors for these relations, but they were not used in experimental fits.

Figures

Slope $\rho_1^2$ vs quadratic coefficient $c_1$ for the $V_1$ form factor.

The $V_{cb}$ matrix element: Tensions in lepton universality

$$R \left( D^{(*)} \right) = \frac{\mathcal{B} \left( B \rightarrow D^{(*)} \tau \nu_{\tau} \right)}{\mathcal{B} \left( B \rightarrow D^{(*)} \ell \nu_{\ell} \right)}$$

- Current $3\sigma$ tension with the SM
The role of Lattice QCD in the exclusive determination

\[ b \bar{u} c \ell \bar{\nu} (\text{Experiment}) = (\text{Known}) \times (\text{CKM}) \times (\text{Had. Matrix El.}) \]

- The lattice allows us to compute hadronic matrix elements from first principles
- Requires experimental data to make full predictions about nature
Heavy quarks in Lattice QCD

Heavy quark treatment in Lattice QCD

- For light quarks \((m_l \lesssim \Lambda_{QCD})\), leading discretization errors \(\sim \alpha_s^k(a \Lambda_{QCD})^n\)
- For heavy quarks \((m_Q > \Lambda_{QCD})\), discretization errors grow as \(\sim \alpha_s^k(am_Q)^n\)
  - In this work \(am_c \sim 0.15 - 0.6\), but \(am_b > 1\)

Need special actions and ETs to describe the bottom quark

- Relativistic HQ actions (this work \(\rightarrow\) FermiLab, Oktay-Kronfeld)
- Non-Relativistic QCD (NRQCD)

If the action is improved enough (HISQ, Domain wall), one can treat the bottom as a light quark

- Highly improved action AND small lattice spacing
- Use unphysical values for \(m_b\) and extrapolate

The discretization errors needn’t disappear \textbf{as long as we keep them under control}\n
Alejandro Vaquero (University of Utah)
Calculating $V_{cb}$ on the lattice

- **Form factors**

\[
\frac{\langle D^*(p_{D^*}, \epsilon') | \mathcal{V}^\mu | \overline{B}(p_B) \rangle}{2 \sqrt{m_B m_{D^*}}} = \frac{1}{2} \epsilon^{\nu^*} \epsilon_\mu^\rho \epsilon_\sigma^\rho v_B^\rho v_{D^*}^\sigma h_{\mathcal{V}}(w)
\]

\[
\frac{\langle D^*(p_{D^*}, \epsilon') | \mathcal{A}^\mu | \overline{B}(p_B) \rangle}{2 \sqrt{m_B m_{D^*}}} = \frac{i}{2} \epsilon^{\nu^*} \left[ g^{\mu\nu} (1 + w) h_{\mathcal{A}_1}(w) - v_B^\nu (v_B^\mu h_{\mathcal{A}_2}(w) + v_{D^*}^\mu h_{\mathcal{A}_3}(w)) \right]
\]

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum.
- The $h_X$ enter in the definition of $\mathcal{F}$.
- We can calculate $h_{\mathcal{A}_{1,2,3,V}}$ directly from the lattice.
Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Helicity amplitudes

$$H_\pm = \sqrt{m_B m_{D^*}} (w + 1) \left( h_{A_1}(w) \pm \sqrt{\frac{w - 1}{w + 1}} h_V(w) \right)$$

$$H_0 = \sqrt{m_B m_{D^*}} (w + 1) m_B \left[ (w - r) h_{A_1}(w) - (w - 1) (r h_{A_2}(w) + h_{A_3}(w)) \right] / \sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2 - 1}{r(1 + r^2 - 2wr)}} \left[ (1 + w) h_{A_1}(w) + (wr - 1) h_{A_2}(w) + (r - w) h_{A_3}(w) \right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w) |\mathcal{F}|^2 = \frac{1 - 2wr + r^2}{12 m_B m_{D^*} (1 - r)^2} \left( H_0^2(w) + H_+^2(w) + H_-^2(w) \right)$$
Calculating $V_{cb}$ on the lattice: Extracting the form factors

- Clever choices of the momentum allow us to isolate the different form factors
- Here $p_{(\perp,\parallel)}$ means $\perp, \parallel$ with respect to the (3D) polarization
- Assuming $(t, x, y, z)$,

$$v^\mu_{D^*} = (w, 0, 0, \sqrt{w^2 - 1}), \quad v^\mu_{\bar{B}} = (1, 0, 0, 0),$$

$$\epsilon_1^\mu = (0, 1, 0, 0), \quad \epsilon_2^\mu = (0, 0, 1, 0), \quad \epsilon_L^\mu = (\sqrt{w^2 - 1}, 0, 0, -w),$$

$$\langle D^*(p_\perp) | V^{1,2} | \bar{B}(0) \rangle = \sqrt{w^2 - 1} h_V(w),$$

$$\langle D^*(p_\perp) | A^{1,2} | \bar{B}(0) \rangle = (1 + w) h_{A_1}(w),$$

$$\langle D^*(p_\parallel) | A^0 | \bar{B}(0) \rangle = \sqrt{w^2 - 1} [ (1 + w) h_{A_1}(w) - h_{A_2}(w) - wh_{A_3}(w)] ,$$

$$\langle D^*(p_\parallel) | A^3 | \bar{B}(0) \rangle = w (1 + w) h_{A_1}(w) - (w^2 - 1) h_{A_3}(w).$$
Calculating $V_{cb}$ on the lattice: Available ensembles

- $N_f = 2 + 1$ staggered asqtad sea quarks
- Heavy quarks use the fermilab action
- Size of the point proportional to the statistics (min 2372, max 15072)
Calculating $V_{cb}$ on the lattice: Two-point functions

$$\langle X(t) \mid X(0) \rangle = \sum_i (-1)^i t \ Z_i^2 \left( e^{-E_i t} + e^{-E_i (T-t)} \right)$$

- Any state that shares quantum numbers with $X$ can be created
  - Use smearing to increase overlap with the desired state
- The staggered formulation adds new (unphysical) particles to the mix
  - It respects a subgroup of chiral symmetry that allows for parity partners
  - The parity partners oscillate with $(-1)^t$ factor and must be fitted

- Fit function
  $$\sum_{i}^{2N-1} (-1)^i (t+1) \ Z_i^2 \left( e^{-E_i t} + e^{-E_i (T-t)} \right)$$

- Consider $N + N$ states ($N$ oscillating, $N$ non-oscillating)
Calculating $V_{cb}$ on the lattice: Two-point functions fits

- Use two different smearings (point and Richardson) at source and sink → four combinations
  - The point sources help with the excited states
  - The smeared sources increase the accuracy of the ground state
- $t_{Min}$ in physical units is common to all the ensembles, $t_{Max}$ is chosen when the points reach 20%-30% error

- Two sets of different data for the $D^*$
  - Momenta $(1, 0, 0)_{\parallel, \perp}$ and $(2, 0, 0)_{\parallel, \perp}$ in lattice units
  - Zero and 8 additional momenta (average $\perp$, $\parallel$)
- Done 2 oscillating $+$ 2 non-oscillating and $3 + 3$ fits to ensure the systematic errors coming from the higher states are under control
Calculating $V_{cb}$ on the lattice: Dispersion relation

- Discretization effects coming from the heavy quark break the dispersion relation.
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha m)$.

$$a^2 E^2(p) = (am_1)^2 + \frac{m_1}{m_2}(pa)^2 + \frac{1}{4} \left[ \frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3} \right] (a^2 p^2)^2 - \frac{am_1 w_4}{3} \sum_{i=1}^{3} (ap_i)^4 + O(p^6)$$

- As long as the discretization errors are under control, this is all right.
- In the Fermilab action we interpret the kinetic mass $am_2$ as the particle mass.
Calculating $V_{cb}$ on the lattice: Three-point functions

\[
\langle Y(0)| \mathcal{W}^\mu(t)|X(T_s)\rangle = \sum_i \sum_j (-1)^i (-1)^j(T_s-t) A_{ij}^{X\rightarrow Y} \times \\
\frac{Z_{Y,i}Z_{X,j}}{2\sqrt{E_{Y,i}E_{X,j}}} e^{-E_{Y,i}t} e^{-E_{X,j}(T_s-t)}
\]

- As before, any state that shares quantum numbers with $X, Y$ can be created
- Oscillating states with weight $(-1)^t$ and $(-1)^{T_s-t}$ also appear in the three-point functions
- A combination of an oscillating state at source and another at the sink introduces an overall offset that depends on the sink $T_s$
- We compute the three-point functions at two consecutive $T_s$ to identify these oscillations
- Traditionally we have employed ratios
  - A clever average removes most of the oscillating behavior
    \[
    \bar{R}(t, T_s) = \frac{1}{2} R(t, T_s) + \frac{1}{4} R(t, T_s + 1) + \frac{1}{4} R(t + 1, T_s + 1)
    \]
  - But this average introduces a bias in some cases
Calculating $V_{cb}$ on the lattice: Three-point functions

\[
\frac{\langle D^*(p_\perp, \varepsilon_{\parallel}) | A | \bar{B}(0) \rangle \langle \bar{B}(0) | A | D^*(p_\perp, \varepsilon_{\parallel}) \rangle}{\langle D^*(0) | V_4 | D^*(0) \rangle \langle \bar{B}(0) | V_4 | \bar{B}(0) \rangle} \rightarrow \left( \frac{1 + w}{2} h_{A_1}(w) \right)^2
\]

- The double ratio was used at $w = 1$ to extract $h_{A_1}$
- At non-zero momentum, there is an exponential dependence on the sink

\[
\frac{C_{D^* \rightarrow B}^A(p_\perp, t, T_s) C_{B^* \rightarrow D^*}^A(p_\perp, t, T_s)}{C_{D^* \rightarrow D^*(0, t, T_s)} V_4^4 C_{B^* \rightarrow B}^V (0, t, T_s)} = \frac{Z_{D^*}^2(p_\perp) M_{D^*}}{Z_{D^*(0)}^2 E_{D^*}} e^{(M_{D^*} - E_{D^*}) T_s} \left( \frac{1 + w}{2} h_{A_1}(w) \right)^2
\]

- Hence the right average to remove the oscillations is

\[
S(t, T_s) = e^{(E_{D^*} - M_{D^*}) T_s} R(t, T_s)
\]

\[
\bar{S}(t, T_s) = \frac{1}{2} S(t, T_s) + \frac{1}{4} S(t, T_s + 1) + \frac{1}{4} S(t + 1, T_s + 1)
\]
Calculating $V_{cb}$ on the lattice: Three-point functions

- Alternatively one can remove the oscillations on each three-point function and then take ratios

$$\bar{C}_{Y \rightarrow X}(t, T_s) = \frac{e^{-E_X t} e^{-E_Y (T_s-t)}}{8} \left[ \frac{C_{X \rightarrow Y}(t, T_s)}{e^{-E_X t} e^{-E_Y (T_s-t)}} + \frac{C_{X \rightarrow Y}(t, T_s + 1)}{e^{-E_X t} e^{-E_Y (T_s-t+1)}} + \frac{2C_{X \rightarrow Y}(t + 1, T_s)}{e^{-E_X (t+1)} e^{-E_Y (T_s-t-1)}} + \frac{2C_{X \rightarrow Y}(t + 1, T_s + 1)}{e^{-E_X (t+1)} e^{-E_Y (T_s-t)}} + \frac{C_{X \rightarrow Y}(t + 2, T_s)}{e^{-E_X (t+2)} e^{-E_Y (T_s-t-2)}} + \frac{C_{X \rightarrow Y}(t + 2, T_s + 1)}{e^{-E_X (t+2)} e^{-E_Y (T_s-t-1)}} \right] \approx$$

$$\approx A_{00} e^{-E_X t} e^{-E_Y (T_s-t)} + (-1)^T_s A_{11} e^{-E'_X t} e^{-E'_Y (T_s-t)} \left( \frac{\Delta E_Y}{2} \right)^2 + \ldots$$
Calculating $V_{cb}$ on the lattice: Three-point functions

- **Alternative approach**
  - Fit the three-point function directly, not the ratio
    - Explicitly takes into account the oscillating states
    - Higher states are also taken into account
  - Compute the ratios using the fitted functions
    The ratios are still needed to remove the non-perturbative part of the renormalization factors
- **Ansatz for 2+2 states:**

$$C_{X \rightarrow Y}(t, T) = \sum_{i}^{2} \sum_{j}^{2} (-1)^{it} (-1)^{j(T_s-t)} A_{X_j \rightarrow Y_i} \times$$

$$Z_{Y,i} Z_{X,j} \frac{e^{-E_{Y,i} t}}{\sqrt{2E_{Y,i}}} \frac{e^{-E_{X,j} (T_s-t)}}{\sqrt{2E_{X,j}}}$$

- Use tight priors for $E_{X,Y}$ and $Z_{X,Y}$ for the ground oscillating and non-oscillating states
- Assumes factorization only for the ground states
Calculating $V_{cb}$ on the lattice: Three-point functions

**Calculated ratios**

\[
\frac{\langle D^*(p) | V | D^*(0) \rangle}{\langle D^*(p) | V_A | D^*(0) \rangle} \rightarrow x_f, \quad w = \frac{1 + x_f^2}{1 - x_f^2}
\]

\[
\frac{\langle D^*(p_\perp, \varepsilon_\parallel) | A | \bar{B}(0) \rangle \langle \bar{B}(0) | A | D^*(p_\perp, \varepsilon_\parallel) \rangle^*}{\langle D^*(0) | V_4 | D^*(0) \rangle \langle \bar{B}(0) | V_4 | \bar{B}(0) \rangle} \rightarrow R_{A_1}, \quad h_{A_1} = \left(1 - x_f^2\right) R_{A_1}^{\frac{1}{2}}
\]

\[
\frac{\langle D^*(p_\perp, \varepsilon_\parallel) | V | \bar{B}(0) \rangle}{\langle D^*(p_\perp, \varepsilon_\parallel) | A | \bar{B}(0) \rangle} \rightarrow X_V, \quad h_V = \frac{2}{\sqrt{w^2 - 1}} R_{A_1} X_V
\]

\[
\frac{\langle D^*(p_\parallel, \varepsilon_\parallel) | A | \bar{B}(0) \rangle}{\langle D^*(p_\perp, \varepsilon_\parallel) | A | \bar{B}(0) \rangle} \rightarrow R_1, \quad h_{A_3} = \frac{2}{w^2 - 1} R_{A_1} (w - R_1)
\]

\[
\frac{\langle D^*(p_\perp, \varepsilon_\parallel) | A_4 | \bar{B}(0) \rangle}{\langle D^*(p_\perp, \varepsilon_\parallel) | A | \bar{B}(0) \rangle} \rightarrow R_0,
\]

\[
h_{A_2} = \frac{2}{w^2 - 1} R_{A_1} \left(w R_1 - \sqrt{w^2 - 1} R_0 - 1\right)
\]

Calculating $V_{cb}$ on the lattice: The recoil parameter $w$

- The recoil parameter can be measured dynamically.
- In the lab frame ($B$ meson at rest):
  \[ w^2 = 1 + v_{D*}^2 \]

- Ratio of three point functions:
  \[ X_f(p) = \frac{\langle D^*(p) | V | D^*(0) \rangle}{\langle D^*(p) | V_4 | D^*(0) \rangle} = \frac{v_{D*}}{w + 1} \]
  \[ w(p) = \frac{1 + x_f^2}{1 - x_f^2} \]

- Errors in the calculation of the renormalization factors are large for $w$.
- It is more advantageous to use the momentum and the dispersion relation:
  \[ w = \frac{E_{D*}}{M_{D*}} = 1 + \frac{p^2}{2M_2} \]
Calculating $V_{cb}$ on the lattice: The recoil parameter $\omega$

- Ratio
- Ratio Matching
- Dispersion Relation
- Momentum
- $E/M_1$

Alejandro Vaquero (University of Utah)

$B \rightarrow D^* \ell \overline{\nu}$ at non-zero recoil

December 16th, 2019
Calculating $V_{cb}$ on the lattice: Current renormalization

- In the coefficients of the terms of our effective theory a dependence arises with the scale (i.e. $a$)
- The renormalization tries to account for the right dependence
- The scheme we employ is called *Mostly non-perturbative renormalization* of results

$$Z_{V^{1,4},A^{1,4}} = \rho_{V^{1,4},A^{1,4}} \times \sqrt{Z_{V_{bb}} Z_{V_{cc}}}$$

- The (relatively large) non-perturbative piece cancels in our ratios
- The (close to one) perturbative piece (matching factor $\rho$) is calculated at one-loop level for $w = 1$, assuming $m_c \approx 0$
- The error for $w \neq 1$ and non-zero charm mass is estimated and added to the factor

This analysis is **blinded** and the blinding happens at the level of the matching factor
Calculating $V_{cb}$ on the lattice: Heavy quark mistuning corrections

- The simulations are run at approximate physical values of $m_c, m_b$
- After the runs the differences between the calculated and the physical masses is corrected non-perturbatively
  - The Fermilab action uses the kinetic mass $m_2$ to compute these corrections
  - $m_1 \rightarrow m_2$ as $a \rightarrow 0$

Correction process

1. For a particular ensemble correlators are computed at different $m_c, m_b$
2. All the ratios are calculated for the new values of the heavy quark masses, and the form factors are extracted
3. The derivative of combinations of the form factors with respect to the heavy quark masses is fitted to a suitable function
4. All the form factors are corrected using these results

Shifts are small in most cases, but add a small correlation among all data points
Calculating $V_{cb}$ on the lattice: The chiral-continuum limit

- Our data represents the form factors at non-zero $a$ and unphysical $m_\pi$
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the $a$ and the $m_\pi$ dependence
- Functional form explicitly known

\[
h_{A_1}(w) = 1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g_{D^*D\pi}^2}{48\pi^2 f_\pi r_1^2} \log_{SU3}(a, m_l, m_s, \Lambda_{QCD}) - \\
\underbrace{\rho^2(w-1) + k(w-1)^2 + c_1 x_l + c_2 x_l^2 + c_a x_a^2 + c_{a_2} x_a^2 + c_{a,m} x_l x_a^2 +}_{w \text{ dependence}} \\
\underbrace{\sum_i z_i(a\Lambda_{QCD})f_i(am_0)}_{\text{NNLO } \chi PT} + \\
\underbrace{c_1 x_l + c_2 x_l^2 + c_a x_a^2 + c_{a_2} x_a^2 + c_{a,m} x_l x_a^2 +}_{\text{NNLO } \chi PT} \\
\underbrace{\sum_i z_i(a\Lambda_{QCD})f_i(am_0)}_{\text{HQET current discretization errors}}
\]

with

\[
x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \quad x_a^2 = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2
\]
Calculating $V_{cb}$ on the lattice: The chiral-continuum limit

- The extrapolation includes some non-trivial effects, like the cusp due to $m_{D^*} - m_D \sim m_\pi$
- Example: $h_{A_1}$ at zero recoil as a function of $m_\pi^2$, NLO fit without discretization errors


- Preliminary blinded results

Results: Chiral-continuum fits

Extrapolation

a = 0.150 fm
a = 0.120 fm
a = 0.090 fm
a = 0.060 fm
a = 0.045 fm

Preliminary Blinded

w

h_{A_1}

h_{V_1}

w

1.00 1.02 1.04 1.06 1.08 1.10 1.12 1.14 1.16
0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5

1.00 1.02 1.04 1.06 1.08 1.10 1.12 1.14 1.16
0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5

Alejandro Vaquero (University of Utah)

B \rightarrow D^* \ell \bar{\nu} at non-zero recoil

December 16th, 2019

32 / 48
Results: Chiral-continuum fits

- Preliminary blinded results
## Analysis: Preliminary error budget

<table>
<thead>
<tr>
<th>Source</th>
<th>$h_V$ (%)</th>
<th>$h_{A_1}$ (%)</th>
<th>$h_{A_2}$ (%)</th>
<th>$h_{A_3}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>1.1</td>
<td>0.4</td>
<td>4.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Isospin effects</td>
<td>0.0</td>
<td>0.0</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$\chi$PT/cont. extrapolation</td>
<td>1.9</td>
<td>0.7</td>
<td>6.3</td>
<td>2.9</td>
</tr>
<tr>
<td>Matching</td>
<td>1.5</td>
<td>0.4</td>
<td>0.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Heavy quark discretization</td>
<td>2.5</td>
<td>1.2</td>
<td>9.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- **Italic** marks errors to be reduced/removed when using HISQ for heavy quarks

- Heavy HISQ would add errors $O(\alpha_s am_b^2)$ and $O((am_b)^4)$ (current errors $O(\alpha_s am_b)$)
Analysis: z-Expansion

- Conformal transformation

\[ z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \]

- Kinematic range \( w_{\text{Min}} = 1 \rightarrow z_{\text{Min}} = 0, \quad w_{\text{Max}} = \frac{1+r^2}{2r} \rightarrow z_{\text{Max}} = \left( \frac{\sqrt{r}-1}{\sqrt{r}+1} \right)^2 \)

- Use BGL expansion

\[ f_X(z) = \frac{1}{\phi_{f_X} B_{f_X}} \sum_j k_j z^j \]

- \( B_{f_X} \) Blaschke factors, includes contributions from the poles in the kinematic range
- \( \phi_{f_X} \) is called outer function and must be computed for each form factor
Analysis: $z$-Expansion

- The expansion is performed on different (more convenient) form factors

\[ g = \frac{h_V(w)}{\sqrt{m_B m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j \]

\[ f = \sqrt{m_B m_{D^*}(1 + w)} h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j \]

\[ \mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j \]

\[ \mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*} \sqrt{w^2 - 1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j \]

- Constraints at small $\mathcal{F}_1(z = 0) = (m_B - m_{D^*}) f(z = 0)$ and large recoil
- BGL (weak) unitarity constraints

\[ \sum_j a_j^2 \leq 1, \quad \sum_j b_j^2 + c_j^2 \leq 1, \quad \sum_j d_j^2 \leq 1 \]
Analysis: ≈ expansion fit procedure

- Several different datasets
  - Our lattice data
  - BaBar BGL fit
  - Belle tagged dataset
  - Belle untagged dataset
  
- Several different fits
  - Lattice form factors only
  - Experimental data only (one fit per dataset)
  - Joint fit lattice + experimental data

- Each dataset is given in a different format, and requires a different amount of processing
- Different fitting strategy per dataset

All the experimental and theoretical correlations are included in all fits
Analysis: $z$ expansion fit procedure

**Lattice data points**
- Generate synthetic data from the chiral-continuum extrapolation
- Fitted directly to the $z$-expansion of the corresponding form factor

**Belle untagged dataset (folded)**
- Two different datasets $e^-$ and $\mu^-$
- Given the fit function, we compute the count prediction on each bin using a similar procedure as in the Belle collaboration

**Belle tagged dataset (unfolded, binned)**
- Integrate the three non-relevant variables in the fit function and fit the result to the data

**BaBar fit results**
- 5 independent parameters + $V_{cb}$
- Generate 5 synthetic datapoints on the $w$ bins and fit them normally
Constraints

- The constraint at zero recoil is used to remove a coefficient of the BGL expansion.
- The constraint at maximum recoil can be imposed by adding a datapoint with small errors.
- The unitarity constraints can be imposed by adding hard cuts.
- In the fits shown, we only impose the constraint at zero recoil, check if the others are satisfied automatically.

How many coefficients?

- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
- Current maximum 3 (each coefficient requires new integrals)
Results: Pure-lattice prediction and joint fit

Separate fits

Joint fit

- Lattice + Belle + BaBar BGL $p$-value = 0.028
- Lattice only BGL $p$-value = 0.503
- Belle untagged BGL $p$-value = 0.16, tagged $p$-value $\approx 1$
Results: Separate fits, angular bins

Alejandro Vaquero (University of Utah)
Results: Joint fit, angular bins

![Graphs showing angular distributions](image)

- **Best fit**
- **Lattice**
- **V_{cb}**
- **Belle untagged e**
- **Belle untagged μ**
- **Belle tagged**
- **BaBar synthetic**

Preliminary
Blinded

Best fit
Lattice
V_{cb}

Belle untagged e
Belle untagged μ
Belle tagged
BaBar synthetic

Preliminary
Blinded

Alejandro Vaquero (University of Utah)
Results: $R(D^*)$

**Separate fits**

- Lattice $\ell = e, \mu$
- Belle untagged $\ell = e^-, \mu^-$
- BaBar
- Lattice $\ell = \tau$
- Belle tagged $\ell = e^-, \mu^-$

**Joint fit**

- $\ell = e^-, \mu^-$
- $\ell = \tau^-$

$\Gamma(B \to D^* \ell \bar{\nu})$

$w$

$1.0 \quad 1.1 \quad 1.2 \quad 1.3 \quad 1.4 \quad 1.5$

Preliminary

Alejandro Vaquero (University of Utah)
Results: Tensions in the BGL coefficients

- The $b_j$ represent the small recoil behavior $\sim h_{A_1}$
- The $c_j$ represent the large recoil behavior $\sim H_0$
Results: Constraints

- Our current lattice results automatically verify the remaining constraints
  - The **unitarity** constraints
  - The large recoil **kinematic** constraint
  - The joint fit shows some tension at large recoil

### Unitarity constraints

<table>
<thead>
<tr>
<th></th>
<th>Lattice only</th>
<th>Joint fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sum_j a_j^2$</td>
<td>0.06(38)</td>
<td>0.24(10)</td>
</tr>
<tr>
<td>$\sum_j b_j^2 + c_j^2$</td>
<td>0.46(60)</td>
<td>0.0031(25)</td>
</tr>
<tr>
<td>$\sum_j d_j^2$</td>
<td>0.15(30)</td>
<td>0.33(90)</td>
</tr>
</tbody>
</table>

### Large recoil constraint

- Preliminary
What to expect

- The outcome of this analysis will provide essential information at small recoil
- The final result might help reduce the tension inclusive/exclusive
- Main source of errors of our form factor seems to be discretization errors
- Preliminary figures for $R(D^*)$ will be released in a few months
- We expect to publish soon, but we won’t disclose results until we deem the analysis ready
  - Our results must meet certain standards of reliability and robustness
  - We only disclose numerical results when we have everything under control
The future

- No date for a future publication, but I aim for no later than spring 2020
  - Check the new 3pt analysis
  - Crosscheck the chiral-continuum extrapolation and the discretization errors
  - Unblinding

- Well established roadmap to reduce errors in our calculation
  - Light HISQ quarks + heavy Fermilab quarks aim to reduce mainly chiral fit errors
  - Light HISQ quarks + heavy HISQ quarks aim to reduce discretization and renormalization errors
  - New runs measure other interesting quantities (i.e. the tensor form factor)

- This roadmap is to be followed in other processes involving other CKM matrix elements
Summary

Lattice and experiment work together to find solutions to the SM dilemmas

Thank you for your attention